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Review Article

Using the Autoregressive Model with External Variables ARX for Forecasting

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Time series forecasting models have witnessed a remarkable development over recent decades as a result of the increasing need to improve the accuracy of forecasting in many scientific and applied fields. The subjective Autoregressive model with external variables (ARX) is one of the important statistical models that combines the impact of historical values of the variable under study and the impact of related external variables, making it an effective tool for forecasting in the fields of Economics, Energy, Environment, Engineering, the health sector and others. This reference article aims to provide a comprehensive scientific review of the ARX model by reviewing its theoretical foundations, mathematical formulation, statistical assumptions, mechanisms for estimating its parameters, and diagnostic procedures, in addition to analyzing published studies that have dealt with its applications in forecasting in recent years, with a critical comparison between it and the most prominent traditional and modern forecasting models. The study adopted the method of systematic review of the scientific literature by analyzing the studies published in the scientific databases, classifying them according to the applied fields, methodologies of model building, methods of performance evaluation, focusing on modern research trends. The results of the review showed that the ARX model achieves high levels of accuracy when external variables with appropriate explanatory ability are available, and it is characterized by Ease of interpretation and low computational complexity compared to many modern models, while its performance declines in cases of non-linear relationships, poor selection of external variables, or the presence of problems such as multiple linear correlation and anomalous values. The review also revealed the existence of research gaps represented by the limited studies that dealt with the development of ARX hybrid models, the lack of research on durable versions of the model, the weak use of machine learning and deep learning techniques to improve its performance, in addition to the scarcity of comprehensive comparisons using multiple databases and unified evaluation metrics. Based on this, the study recommends directing future research towards the development of hybrid models combining ARX, machine learning and deep learning technologies, taking advantage of modern variable selection methods, and developing models capable of dealing with nonlinear and high-dimensional data and real-time forecasting in dynamic environments, thereby contributing to improving prediction accuracy and increasing the efficiency of Model applications in various fields.

Keywords: *Autoregressive model with external variables (ARX), time series, forecasting, external variables, literature review, evaluation of models, machine learning, hybrid models*

Introduction

Forecasting is one of the most important scientific tools used to support strategic planning and decision-making in various economic, engineering, environmental, medical and social fields. government institutions and the private sector rely on forecasting results to prepare future plans, manage resources, reduce risks and improve the efficiency of operational processes. The importance of forecasting models has increased in recent years with the accelerated development of data collection technologies and the emergence of big data, which has imposed the need to develop models that are better able to interpret complex time patterns and achieve higher levels of predictive accuracy (Box et al., 2016; Hamilton, 2020). The field of time series analysis has witnessed a remarkable development since the emergence of classical statistical models that relied on the historical values of the variable under study, where the box–Jenkins methodology constituted a major turning point in the construction of time series models by providing a methodological framework that includes the stages of determining the model, estimating its parameters and diagnosing it before using it in forecasting. This methodology contributed to the development of Autoregressive models (AR), moving average models (MA), then compact models (ARMA), and finally Autoregressive models and integrated moving averages (ARIMA), which have become one of the most widely used models in statistical, economic and engineering applications (Box et al., 2016; Jenkins, 2014).

Despite the great success achieved by these models, they rely mainly on the internal information of the time series, that is, on the previous values of the variable itself, without taking into account the influence of external factors that may have a direct role in explaining the behavior of the target variable. In many real-world applications, such as forecasting energy consumption, temperatures, inflation, exchange rates, and industrial production, future changes can not only be explained based on historical data, but are also influenced by a set of related external variables. Hence, the need arose to develop models that allow integrating this additional information into the modeling process, which led to the emergence of the autoregressive with Exogenous variables (ARX) autoregressive model, which combines dependence on the previous values of the dependent variable and the use of exogenous explanatory variables at the same time, which enhances the explanatory ability and raises the accuracy of forecasting in many practical applications. Time series models differ among themselves in terms of mathematical structure and the nature of the data they process. The autoregressive (AR) model relies on the previous values of the dependent variable only, while the ARMA model adds the moving averages component to represent the correlation found in random errors when dealing with stable series. The ARIMA model includes differential operations to process unstable chains and convert them into stable chains before building the model. In contrast, the ARX model is characterized by the introduction of external variables that represent additional information that may have a direct impact on the variable under study, which gives it a better ability to explain multifactorial phenomena compared to traditional models that rely only on internal information (Hamilton, 1994; Ljung, 1999). External variables are one of the key elements that autoregressive ARX model from the rest of the traditional time series models, as these variables represent additional sources of information that help explain future changes to the dependent variable. For example, temperature, humidity and wind speed can be used as external variables in forecasting models of energy consumption, oil prices, interest rates or inflation rates can be used when forecasting economic indicators, while precipitation, humidity or climatic variables can be introduced into environmental and hydrological forecasting models. The selection of suitable external variables improves the quality of the model and reduces prediction errors, while the selection of inappropriate variables may lead to a decrease in predictive performance or increase the complexity of the model without real benefit. This reference article aims to provide a comprehensive scientific review of the autonomous Autoregressive model with external variables (ARX), by reviewing the theoretical basis of the model, its mathematical formulation, statistical assumptions, methods for estimating its parameters, methods for verifying its suitability, in addition to reviewing and analyzing the applied studies that used it in the fields of energy, economics, environment, engineering, the health sector and others, with a critical comparison between it and the most prominent traditional and modern forecasting models, highlighting its advantages and applied limitations. The scientific contribution of this study is that it is not limited to presenting previous studies, but provides a critical and systematic analysis of modern research trends related to the ARX model, classifying studies according to application areas, model building methods and performance evaluation metrics, identifying research gaps that still need further study, especially with regard to the development of hybrid models, integrating machine learning and deep learning techniques with the ARX model, improving its ability to handle nonlinear and high-dimensional data and dynamic environments, thereby contributing to directing future research towards developing more efficient and accurate models in forecasting (Ljung, 1999).

Fundamentals of Time series Forecasting

A time series is defined as a set of observations or measurements that are recorded for a particular variable in regular chronological order, so that each observation represents the value of the variable at a specific point in time. The analysis of time series is aimed at studying the patterns inherent in the data, such as the general trend, seasonal, cyclical and random fluctuations, with the aim of understanding the behavior of the phenomenon under study and building models capable of predicting future values with a high degree of accuracy. Time series data are characterized by the presence of a time dependence between successive observations, which distinguishes them from cross-sectional data in which the independence of observations is assumed. Time series have become one of the most widely used types of data in economics, finance, engineering, energy, medicine and environmental sciences, as a result of advances in data collection and storage technologies and the increasing need to support future decisions based on reliable historical data (Box et al., 2016; Brockwell & Davis, 2016; Hyndman & Athanasopoulos, 2021). The time series usually consists of four main components that together contribute to shaping the behavior of data over time. The first component is the general trend, which expresses the long-term movement of data, whether increasing or decreasing as a result of economic, social or technological factors. The second component is seasonality, which refers to regular changes that are repeated over fixed periods of time such as months, seasons or days of the week. The third component is the cyclical Component, which are fluctuations that extend for longer periods of seasonality and are often associated with economic or industrial cycles without having a fixed repetition period. As for the last component, it is the random or irregular component (Irregular or Random Component), which represents unexpected changes resulting from exceptional events or factors that are difficult to model. The analysis of these components helps to choose the appropriate prediction model and improve its accuracy, as most time series models rely on characterizing these components correctly before starting to build the statistical model (Hyndman & Athanasopoulos, 2021; Box et al., 2016). Forecasting can be classified according to several criteria, and the temporal classification is one of the most common, as it is divided into short-term forecasting that extends from several hours to several months and is used in operational planning and inventory management, medium-term forecasting that usually covers several months to a few years and is used in production planning and investments, and long-term forecasting that extends for many years and is used in strategic planning and policy making. Forecasting methods can also be classified into qualitative Forecasting methods based on experience and personal estimation in case of data scarcity, and quantitative Forecasting methods based on mathematical and statistical models and machine learning when historical data are available. AR, ARMA, and ARX models are among the quantitative forecasting methods based on time series analysis, which aim to exploit historical information to build models capable of predicting the future with minimal error (Petropoulos et al., 2022; Hyndman & Athanasopoulos, 2021).

Stability is one of the most basic assumptions in most linear time series models, as it indicates that the statistical properties of the series, such as mean, variance and self-variability, remain constant over time and do not depend on the location of the time period. Stability is divided into strict stability (Strict Stationarity) and weak stability (Weak or Covariance Stationarity), and most statistical models rely on the second type for being more suitable for practical applications. If the stability is not achieved, the estimation results may become misleading and the models lose their predictive power, so procedures such as logarithmic conversion, first diff or seasonal diff are used to achieve stability before the model is built. Well–known statistical tests, such as the extended Dickey-Fuller test (ADF) and the KPSS test, are also used to verify the stability of the time series before starting the modeling process (Hamilton, 1994; Hyndman & Athanasopoulos, 2021). Autocorrelation refers to the degree of relationship between the values of the current time series and their previous values at different lags periods, and is one of the most important characteristics on which the analysis of time series depends. Autocorrelation is measured using the autocorrelation Function (ACF), which shows the strength and direction of the relationship between current and previous observations. This function helps to detect the presence of trend, seasonality or instability, and is a key tool in choosing the appropriate model within the box–Jenkins methodology. For example, a slow decrease in the coefficients of autocorrelation leads to the possibility of instability, while a rapid decrease indicates that the chain is stable and can be modeled using ARMA, ARIMA or ARX models (Box et al., 2016; Brockwell & Davis, 2016).

Partial autocorrelation represents the direct relationship between two values in the time series at a certain deceleration interval after removing the influence of all intermediate intervals located between them. This concept is presented through the Partial autocorrelation Function (PACF), which is an essential tool in determining the rank of the autocorrelation model (AR), since the first significant values of the PACF function indicate the number of appropriate slowdowns of the model. The combined analysis of ACF and PACF is used within the box–Jenkins methodology to determine the most suitable model, as their combination helps to distinguish between AR, MA and Arma models, and also forms an important basis in the construction of ARIMA and ARX models, due to the dependence of these models on the temporal structure of the series before the integration of external variables in the forecasting process (Box et al., 2016; Hamilton, 1994; Brockwell & Davis, 2016). The autoregressive with Exogenous Variables, ARX, is one of the linear models widely used in time series analysis and forecasting, combining the characteristics of autoregressive models and the influence of exogenous explanatory variables within a unified mathematical framework. This model is based on the premise that the current value of the dependent variable is influenced not only by its previous values, but also by a set of external variables that possess explanatory power for the behavior of the time series. Thus, the ARX model is a natural extension of the traditional self-autoreressive model (AR) and adds external information that enables improving the accuracy of forecasts and interpretations of influences (Ljung, 1999; Söderström & Stoica, 1989). The ARX model has gained great importance in multiple fields, such as control engineering, energy demand forecasting, Industrial Systems Analysis, economics, finance, environmental forecasting, due to the simplicity of its mathematical structure and the ease of estimating its parameters compared to more complex models. The model also allows the inclusion of one or several external variables with the possibility of using different lags for each variable, which provides great flexibility in representing dynamic relationships between variables (Ljung, 1999; Billings, 2013).

Theoretically, the ARX model is classified among Linear dynamic systems models and is widely used in the field of system identification, as it is one of the most common mathematical structures for modeling the relationship between inputs and outputs, especially when the external variables are measurable and can be obtained before the prediction process (Ljung, 1999). This representation is widely used in the literature of systems definition and stability analysis, and also facilitates the derivation of model properties and analysis of its dynamic response (Ljung, 1999; Söderström & Stoica, 1989). This representation is the mathematical basis from which most parameter estimation methods are derived, such as the ordinary Least squares method (Ordinary Least Squares, OLS) and the maximum Likelihood Estimation method (MLE), and is also used in analyzing the properties of estimators and studying their consistency, efficiency and statistical neutrality (Hamilton, 1994; Ljung, 1999).

In general, the matrix representation provides a unified framework for dealing with ARX models no matter how many external variables or slowdowns are used, which makes it the standard method in most statistical software such as MATLAB, R, Python, yeviews, Gretl, as well as system definition applications in engineering and control. The selection of external variables is one of the most important stages in building a self-autoregressive model with external variables (ARX), because the quality of the model depends not only on determining the rank of the subjective part, but also largely on the selection of explanatory variables that have a real relationship with the dependent variable. Relying on non-influential variables or including a large number of unnecessary variables leads to increased Model complexity, lower estimation efficiency, and a high probability of overfitting, while neglecting influential variables leads to model bias and reduced prediction accuracy. Therefore, the choice of external variables should be based on theoretical knowledge of the nature of the studied phenomenon, supported by statistical and economic evidence and the results of previous studies, taking into account the availability, quality and timeliness of data (Ljung, 1999; Hamilton, 1994). In practical applications, the nature of external variables varies depending on the field of study; in energy models, they may include temperature, humidity and wind speed, while in economic studies they include interest rates, inflation, oil prices and the exchange rate, and in environmental applications they may include the amount of precipitation or the concentration of air pollutants.

Therefore, the scientific selection of external variables is an essential step to improve the explanatory ability of the model and increase the accuracy of forecasting. The problem of multilinearity is one of the most common problems when dealing with external variables, and it occurs when there is a strong correlation relationship between two or more explanatory variables, which leads to inflation and instability of the variations of autoreressive estimators, and therefore the difficulty of interpreting the effect of each variable independently. Multilinearity usually does not bias the estimates of the model, however, it reduces their efficiency and makes the results of statistical tests less reliable. To detect this problem, several indicators are used, the most important of which are the coefficient of variation inflation Factor (VIF), the coefficient of permittivity (Tolerance), and the correlation matrix between variables. A high VIF value indicates a high correlation between external variables, which necessitates deleting some variables, merging them, using dimensionality reduction techniques such as Principal Component Analysis (PCA), or applying regular autoreressive methods (Regularization) when needed. This step is especially important in multivariate ARX models, since a strong correlation between external variables may lead to poor predictive performance of the model. Determining lag Selection is one of the key elements in building an ARX model, as it involves not only choosing the rank of the subjective part of the model, but also determining the number of appropriate slowdowns for each external variable. The choice of a small number of slowdowns leads to the neglect of important time effects, while the choice of a large number of them leads to an increase in the number of parameters and a weakening of the predictive ability of the model as a result of over-matching.

Several statistical criteria are used to select optimal slowdowns, most notably the akaiki information criterion (AIC), the Bayes information criterion (BIC), the Hannan–Quinn criterion (HQIC), as well as the final prediction error criterion (Final Prediction Error, FPE). The researcher should also take into account the theoretical background of the studied phenomenon, since some variables may have an immediate effect, while the effects of other variables appear after several periods of time. Studies show that the optimal selection of decelerations directly contributes to improving the accuracy of Dynamic models and tests of causality. Cross-Correlation Function (CCF) analysis is used to explore the temporal relationship between the dependent variable and external variables at different slowdowns. This analysis makes it possible to determine the time period at which the strongest correlation between the two variables appears, which helps in choosing the appropriate slowdowns of external variables before building the ARX model. Correlation analysis should be carried out after confirming the stability of time series and removing trend or seasonality, when necessary, since the presence of common trends may lead to the appearance of false correlations that do not reflect a real relationship between the variables.

CCF analysis is therefore an important exploratory tool, but it is not enough on its own to prove causality, it should be supported by more advanced tests such as Granger causality tests. The selection of characterizes has become one of the most important parts of the forecasting model when dealing with a large number of external variables. The selection of characteristics is aimed at identifying the most influential variables in the dependent variable, excluding insignificant or repetitive ones, thereby improving the predictive performance and reducing the computational complexity of the model.

Methods for selecting properties vary between filter methods (Filter Methods), which rely on correlation coefficients or mutual information, wrapping methods (Wrapper Methods), which rely on evaluating the performance of the model, and embedded methods (Embedded Methods) such as LASSO and elastic Net, which carry out the selection process during model estimation. In recent years, many studies have tended to integrate feature selection techniques with ARX models to improve performance when working with high-dimensional data, which is a promising research direction in modern forecasting applications. A statistical correlation between variables is not enough to justify their inclusion in the ARX model, rather, a predictive relationship between variables should be checked using causality tests, and the Granger causality Test is one of the most commonly used tests for this purpose. This test is based on examining whether the previous values of the external variable contribute to improving the prediction of the dependent variable compared to a model that relies on the previous values of the dependent variable only. It is important to emphasize that Granger causality expresses predictive causality, and does not necessarily imply a real causal relationship in a philosophical or empirical sense. The test results also depend heavily on the choice of the appropriate number of slowdowns, the stability of time series, and the inclusion of all relevant variables, since the omission of influential variables can lead to misleading conclusions. That is why recent studies recommend the use of causality tests in integration with the criteria for choosing decelerations and economic or physical analysis of the studied phenomenon when constructing ARX models.

Forecasting Procedure using ARX

The construction of a self-autoregressive model with external variables (ARX) is a systematic process consisting of a set of sequential stages, starting with data collection and ending with an assessment of the accuracy of the forecast. The success of the model depends on the implementation of each stage according to sound statistical bases, as any defect in one of these stages may negatively affect the quality of the model and the efficiency of forecasting. The steps for constructing the ARX model correspond to the general methodology for constructing time series models, which includes defining the model, estimating its parameters, examining its suitability, and then using it in forecasting. The data collection stage is the first step in building the ARX model, as the time series of the dependent variable is obtained, as well as the external variables expected to affect it. The data should be measured at regular time intervals, be accurate, consistent and complete, making sure that the units of time correspond between different variables. Also, a sufficient period of time should be chosen to represent all dynamic patterns, such as a general trend, seasonality or economic cycles, since a short time series may lead to a poor ability to accurately estimate the parameters. The literature indicates that data quality is one of the most important factors affecting the quality of predictive models, regardless of the level of complexity of the model used. The next step after data collection is to clean the data for statistical analysis. This includes removing missing, anomalous values, correcting measurement errors, input errors, and consolidating time units when data are from more than one source. Some applications may require transformations such as natural logarithm, standardization, or normalization to reduce variability and improve data properties. It should also be checked that there are no sudden structural changes that may affect the stability of the model, since the quality of the input data is a prerequisite for obtaining reliable estimates and low prediction errors.

Stability is one of the most important fundamental assumptions in linear ARX models, and therefore it should be checked before starting to estimate the model. By stability is meant the constancy of the statistical properties of a time series, such as mean, variance and self-variability, over time. Multiple tests are used to verify this, the most famous of which are the augmented Dickey–Fuller test (ADF), the Phillips–Perron test (PP), and the KPSS test. In the case of chain instability, the operations of difference, logarithmic transformation or removal of the general direction are used until the conditions for constructing the model are met. It is also recommended to conduct the same tests on external variables to ensure their compliance with the assumptions of the model. After confirming the stability of the data, the ranks of the model are determined, they include the rank of the subjective part (na), the rank of the part of the external variables (nb), the delay time (nk). At this stage, the analysis of the autocorrelation (ACF) and partial autocorrelation (PACF) functions is used, along with information standards such as the akaiki criterion (AIC), the Bayes criterion (BIC), the Hannan–Quinn criterion (HQIC), and the final error criterion for forecasting (FPE). Proper selection of slowdowns contributes to improved predictive performance and reduced computational complexity, while choosing inappropriate ranks may lead to poor model accuracy or over-matching. As we have already determined the structure of the model, we need to estimate the parameters using one of the appropriate methods. The least squares method (Ordinary Least Squares, OLS) is the most commonly used method for estimating parameters in ARX models due to its simplicity and effectiveness provided that the basic statistical assumptions are met. For some models, such as models where the error distribution is not independent or follows a certain distribution, we can use the maximum Likelihood Estimation (MLE) method. Modern algorithms based on Bayesian estimation or numerical optimization are also available for more complex models. In matlab programs and system definition tools, the estimator of an ARX model is generally estimated using the least squares method after determining the ranks of the model.

After the parameters are estimated, a set of diagnostic tests is performed to check that the model is valid for use in forecasting. These tests include analyzing the buffers to verify that they represent white Noise (White Noise), using the Ljung–Box test to detect residual self-correlation, as well as contrast smoothing tests, examining the normal distribution of buffers, checking the constancy of parameters over time. If the model does not pass these tests, the grading process, the selection of external variables or the re-estimation of the model are repeated until all the necessary statistical assumptions are fulfilled. This stage is a key component of the box–Jenkins methodology for constructing time series models. The last stage represents the evaluation of the model's performance by comparing the predicted values with the actual ones using a set of accuracy measures. The most commonly used measures are the mean square error (MSE), the root mean square error (RMSE), the mean absolute error (MAE), the mean absolute error ratio (MAPE), as well as the Theil's U coefficient and the Diebold–Mariano test (Diebold–Mariano Test) when comparing more than one model. It is also recommended to divide the data into a training Set and a testing Set, or use appropriate cross-validation methods for time series, in order to obtain a realistic assessment of the model's ability to predict outside the sample. This stage is the basis on which the final judgment on the suitability of the ARX model in comparison with competing models is built. Forecasting accuracy measures are one of the most important tools used in evaluating the performance of time series models, as they are used to compare different models and choose the one that is most capable of predicting future values. These scales are based on the analysis of differences between actual and predicted values, and each scale differs in its sensitivity to outliers, its dependence on the data scale, and the comparability of different time series. Modern literature therefore recommends using more than one scale when evaluating models, and not relying on only one, since each scale reflects a certain aspect of the quality of forecasting.

Forecast Accuracy Measures

RMSE is one of the most widely used metrics for evaluating forecast accuracy, it measures the average magnitude of forecast errors while giving more weight to large errors due to their squaring, therefore it is suitable for applications where large errors are more important than small ones. MAE measures the average absolute value of forecast errors without squaring them, therefore it is less affected by anomalous values compared to RMSE. MAPE expresses the error in the form of a percentage, therefore it is widely used when comparing the performance of models on data of different scales. He developed MAPE to address some of the problems of MAPE, especially when the actual values are small or close to zero. Theil's U statistic is used to compare the model's performance with a simple reference model, often a random walk model or a naive prediction model. When comparing two forecasting models, it is not enough to rely on the difference of RMSE or MAE values, since this difference may be caused by random variability. The Diebold–Mariano test (DM Test) is therefore used to statistically verify that there is a significant difference between the accuracy of two prediction models. The test is calculated based on the average of the differences of the loss functions (Loss Differential), then the test statistic value is extracted and the level of statistical significance is compared with the predetermined value (usually 0.05). The Diebold-Mariano test is the standard in the forecasting literature when comparing models, because it not only compares error values, but also determines whether the differences in performance are statistically significant. Therefore, most recent studies recommend using it when evaluating the ARX model against models such as ARIMA, NARX, or machine learning models. The scale depends on the unit of measurement sensitive to outliers suitable for comparison of different series.

Literature Review

The period from the Eighties of the last century until 2005 witnessed the stage of scientific establishment of the autonomous autoreressive model with external variables (ARX), as its roots originated from the research of the definition of dynamic systems (System Identification), which aimed to build mathematical models that describe the relationship between inputs and outputs based on the observed data. Although the actual beginnings date back to the pioneering work of Åström and Bohlin (1965), which laid the foundation for prediction error methods (Prediction Error Methods) in estimating Dynamic models, the comprehensive review by Åström and weykhoff (1971) contributed to the consolidation of the concept of defining systems and clarifying the basic principles of choosing the model structure and estimating its parameters, which paved the way for the emergence of ARX models as one of the most dynamics. Ljung and Söderström (1983) showed that the linear structure of the model allows the use of recursive Least Squares algorithms for efficient parameter estimation, which made these models suitable for applications that require constant updating of parameters. In the same context, Söderström and Stoica (1989) presented an advanced mathematical treatment of the properties of ARX model estimators, focusing on consistency, efficiency, bias and model rank selection, which promoted their use in industrial control applications and engineering systems.

By the end of the nineties, this field witnessed a remarkable development with the publication of the second edition of the book Ljung (1999), which is the main reference in the definition of systems, where the ARX model was comprehensively dealt with in terms of mathematical formulation, methods for estimating parameters, choosing the ranks of the model, checking the validity of the remainder, in addition to its practical applications using MATLAB. This reference has become the theoretical basis on which most subsequent research has relied in the development of ARX models and their applications in the fields of engineering, economics and ecology. At the beginning of the new millennium, research attention shifted to the study of the statistical performance of the model in practical applications. Weyer (2000) showed that the characteristics of ARX estimators are affected by the sample size and the length of the time series, and that small samples may lead to increased parameter variability and reduced prediction accuracy, while Qin, Lin and Ljung (2005) proposed developments on subsystem identification algorithms, showing that ARX models represent the starting point for many modern identification methods, and that improving the choice of model structure reduces bias and raises estimation efficiency, especially in closed-loop systems (closed-loop systems). As a result, by 2005 ARX models had become one of the most widely used in control and systems definition applications, and formed the basis from which subsequent developments, including hybrid models, Bayesian models, and machine learning models based on external variables, were launched.

The autoregressive model with external variables (ARX) is one of the effective linear models in predicting time series when the influence of external variables is obvious and can be represented linearly.it surpasses the autoregressive model (AR), ARMA and ARIMA models in cases where explanatory variables directly affect the time series, while the ARIMAX model is an extension of ARIMA by adding external variables, which makes it more suitable for unstable data that require making distinctions. ARX is also similar to Transfer Function and dynamic autoregressive models in employing external variables, however, these models provide greater flexibility in representing complex dynamic effects. In contrast, VAR models are more suitable when there are several interrelated internal variables, while the state Space and Kalman filter models are distinguished by their ability to cope with dynamic systems, unobserved variables and time-varying data. With the significant development of artificial intelligence technologies, machine learning and deep learning models such as Neural Networks, LSTM, GRU, random Forest, xgboost and support vector Autoregressive (SVR) have proven their ability to model complex nonlinear relationships and achieve high predictive accuracy, but they often require large volumes of data and are characterized by low interpretability compared to the ARX model. The model developed by Professor Peter P. Pritzker has also proved useful in estimating time series with multiple trends and seasonality, particularly in the commercial field. Recent research indicates that the optimal model is chosen depending on the nature of the data, the degree of linearity, the availability of external variables, the number of data points, and the need for interpretation. The ARX model remains a suitable model when requirements are for simplicity, interpretation, and efficiency in using external information, while hybrid models combining ARX and machine learning excel in many recent applications (Box et al., 2015; Hyndman & Athanasopoulos, 2021; Billings, 2013; Breiman, 2001; Hochreiter & Schmidhuber, 1997; Chen & Guestrin, 2016).

Advantages of the ARX Model

The autoregressive model with external variables (ARX) has a number of advantages that have made it one of the most widely used linear models in forecasting and systems definition applications. One of its most important advantages is the simplicity of its mathematical structure and the ease of interpretation of its parameters, as it can be estimated using the usual least squares method (OLS) efficiently and without the need for complex algorithms, which contributes to reducing computational time and ease of application in practical environments. The model is also characterized by fast estimation and low computational complexity compared to nonlinear models and deep learning models, in addition to its suitability for relatively small samples when the assumptions of the model are fulfilled. The main advantages are also its ability to integrate external variables into the forecasting process, which improves the accuracy of forecasting when these variables have a direct impact on the time series, as well as its ease of integration with more advanced models such as state Space Models and machine learning techniques to develop hybrid models with higher predictive performance (Ljung, 1999; Söderström & Stoica, 1989; Billings, 2013; Hyndman & Athanasopoulos, 2021).

In recent years, there has been an increasing trend towards the development of hybrid models that combine the autoregressive model with external variables (ARX), machine learning and deep learning techniques in order to improve prediction accuracy and address nonlinear relationships that a linear model may not be able to adequately represent. The ARX-LSTM model was used to take advantage of the ability of long-term short-term memory networks (LSTM) to learn long-term time dependencies while retaining the ability of ARX to represent the influence of external variables, ARX-CNN was used to derive local patterns from temporal data, and ARX-GRU to provide predictive performance similar to

LSTM with lower computational complexity. ARX is also integrated with Random Forest and xgboost algorithms to improve variable selection and nonlinear modeling, while recent studies have tended to integrate it with attention models to enhance the ability of the model to focus on the variables and time periods that most affect the forecasting process. The results of most studies indicate that these hybrid models achieve higher accuracy than the traditional ARX model, especially in applications related to Energy, Economics, intelligent systems, while maintaining the possibility of taking advantage of the information provided by external variables (Hochreiter & Schmidhuber, 1997; Breiman, 2001; Chen & Guestrin, 2016; Vaswani et al., 2017; Goodfellow et al., 2016).

Future Research Directions

Most recent research suggests that the future direction of the autoregressive model with external variables (ARX) development is moving towards its integration with modern technologies in artificial intelligence and machine learning to improve prediction accuracy and model efficiency. For example, explainable AI models are developing to make predictions more transparent, and hybrid Forecasting Models using ARX in combination with deep learning techniques such as LSTM, GRU, and transformers to make the most of their capabilities to represent complex nonlinear relationships are being developed. Development of Bayesian ARX models for uncertainty in estimation and probabilistic Forecasting to provide a range of confidence interval along with expected value estimates. Promising trends include continuous learning (Online Learning) and real time forecasting (Real-time Forecasting) so that the model can be updated in real-time as the data flows by the model, edge Computing for reducing the response time to IoT applications, and linking ARX models with digital Twins to assist in monitoring, simulation, and decision-making in intelligent industrial systems. These trends should help to expand the use of the ARX model in smart industry, renewable energy, smart cities, and Cyber-Phys with its accuracy, interpretation, and adaptability to change (Goodfellow et al., 2016; Vaswani et al., 2017; Billings, 2013; Hyndman & Athanasopoulos, 2021; Ribeiro et al., 2016).

Conclusions

The self-autoresponsive model with external variables (ARX) is one of the Effective Statistical models to predict time series due to the ability to combine past values of the dependent variable and external influences in a simple, easy to understand mathematical format. The literature has shown that the model has developed from its origins in systems definition and extension to energy, economics, finance, environment, hydrology, medicine, and intelligent industrial systems where it proved to be effective, especially for linear or semi-linear data. The review also reported that the ARX model performs best when the data are of good quality, external variables are selected appropriately, time lags are identified, and the statistical assumptions are tested. Arx outperforms most linear models when the relevant variables are observed to have an influence, while hybrid models with ARX combined with machine learning and deep learning perform best when dealing with non-linear or complex data. On top of that, this study showed that modern research is not limited to improving the old model, but has been increasingly utilizing the ARX model with artificial intelligence, probabilistic forecasting, Bayesian models, continuous learning, and intelligent systems, because it continues to be one of the main models on which to build more accurate, flexible, and interpretable prediction systems. Therefore, the ARX model is still considered an important model of science and practice when it is employed in a hybrid way that uses the benefits of statistical models and recent artificial intelligence technologies.

Recommendations

Based on this review, it is recommended to use the ARX model in forecasting situations where external variables have a strong influence, and choose these variables based on the appropriate statistical methods, tests of causality and correlation analyses. It is also recommended to assess the model with not only one measure of forecast accuracy but also multiple measures, such as RMSE, MAEYE, Mase, Theil's U and Diebold–Mariano test. The study also recommended developing hybrid models that combine ARX and machine learning and deep learning technologies, such as LSTM, GRU and xgboost to take advantage of their ability to model nonlinear relations with the interpretability of the ARX model. Also, it was recommended to use more Bayesian models, probabilistic forecasting and continuous learning (Online Learning) to provide real-time prediction applications in the areas of renewable energy, smart cities, the Internet of things, and industrial systems. Finally, it recommended more applied research on comparing the performance of the ARX model with modern models using different databases and different geographic locations, in particular Big Data, high dimensions, and structural changes, which will lead to more efficient and reliable models and enhance the potential use of the ARX model in the future in the field of digital transformation and artificial intelligence.

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Conflict of Interest

The authors declare that there are no conflicts of interest regarding the publication of this manuscript.

Ethical Approval

Ethical approval was not required for this study as it did not involve human participants, personal data.

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